AD-A163 752 FORTRAN PROGRAM FOR VIBRATION AND SOUND RADIATION OF SPHERICAL SHELL(U) ADMIRALTY RESEARCH ESTABLISHMENT PORTLAND (ENGLAND) J H JAMES JAN 86 ARE/TM-(N1)-86501 1/1

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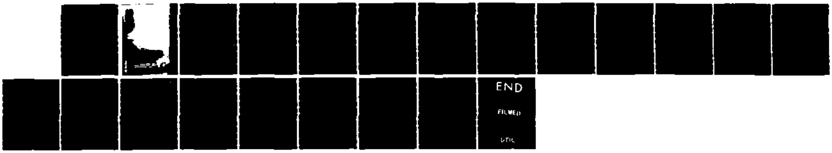

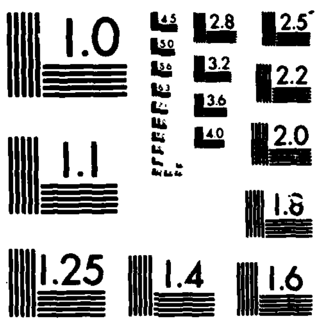
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APPENDIX A

PREVIOUS PAGE

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ARE-TM(N1)86501

FORTRAN PROGRAM FOR VIBRATION AND SOUND RADIATION OF SPHERICAL SHELL

BY

J H JAMES

Summary

Mathematical formulae and a Fortran program listing are given for calculating the response and sound radiation due to prescribed time-harmonic excitation. The program is useful for bench-mark studies. Numerical results illustrate the applicability of the various "DAA" forms of the exterior fluid loading term.

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22 pages
3 figures

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1. INTRODUCTION

The submerged spherical shell is the only finite elastic system for which closed-form expressions are available for its vibration and sound radiation. Its simplicity makes it a useful bench-mark problem against which purely numerical formulations may be tested. Much of the theoretical work on the dynamics and acoustics of the spherical shell is contained in the standard text [1] on fluid-structure interaction, authored by Junger and Feit. The vibration of a point force excited spherical shell in air and in water has been studied by Wong & Hayek [2] who found good agreement between the resonant frequencies determined by theory and experiment. James [3] has presented target strength spectra and intensity vector plots which show the considerable distortion of the sound field in the vicinity of a spherical shell, when it is excited at a resonant frequency. Ansley & Skelton [4] have studied the scattering of sound by multilayered spherical shells using the exact theory of linear elasticity: some of their numerical work (not published) demonstrates that the shell theory approximation is valid for radiated sound calculations over a wide range of frequency and thickness to radius ratios.

Recent theoretical research in fluid-structure interaction problems has developed approximations which avoid the lengthy computations required by the Helmholtz integral equation. These theories, which are collectively known by the name 'doubly asymptotic approximations (DAA's)', converge correctly at low and high frequencies to the 'added mass' and 'plane wave' approximations, respectively. They have been obtained by Geers [5] and summarized by Geers & Felippa [6].

The vibration and sound radiation of a spherical shell is the standard bench-mark problem for testing the performance of the various DAA's, both analytically and numerically. Geers & Felippa [6] have given closed-form expressions for the DAA's of the submerged spherical shell, and have presented numerical results which show that the second order approximations are adequate, except at frequencies close to resonances. Their findings have been reinforced by the numerical calculations of Huang & Wang [7].

The numerical studies of Geers & Felippa are limited to frequency response plots of the shell's velocity and surface pressure in the individual terms (n=0,3) of their Legendre series expansions. The numerical comparisons of Huang & Wang are based on polar plots of the far-field sound radiation, at selected frequencies: their comparisons could be misleading close to a resonant frequency because small differences in the resonant frequencies, which in themselves are of no practical importance, could cause unduly large differences between the sound pressure levels at a selected frequency. Thus, in addition to providing a computer program which can be used for bench-mark studies, it is the purpose of this Memorandum to extend the range of numerical tests of the applicability of the DAA's to the spherical shell problem.

2. RÉSUMÉ OF MATHEMATICS

The radial and tangential displacements of the shell's surface are expressed as a series of Legendre polynomials of argument $\mu=\cos\theta$, viz.

$$U(\theta) = \sum_{n=0}^{\infty} (1-\mu^2)^{1/2} U_n P_n'(\mu) \tag{1}$$

$$W(\theta) = \sum_{n=0}^{\infty} W_n P_n(\mu) \tag{2}$$

whose amplitudes are obtained from the matrix relation

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \begin{vmatrix} U_n \\ W_n \end{vmatrix} = \begin{vmatrix} 0 \\ F_n \end{vmatrix} \tag{3}$$

where for coupled membrane and bending theory in which effects due to rotary inertia and transverse shear are neglected

$$\begin{aligned} a_{11} &= E_1(1+\beta^2)(\sigma+\lambda_n-1)-\omega^2\rho_s h \\ a_{12} &= E_1\beta^2(\sigma+\lambda_n-1)+E_1(1+\sigma) \\ a_{21} &= \lambda_n a_{12} \\ a_{22} &= E_1\beta^2\lambda_n(\sigma+\lambda_n-1)+2E_1(1+\sigma)-\omega^2\rho_s h \\ &\quad +\rho_2 c_2\omega h_n(k_2a)/h_n'(k_2a) - \rho_1 c_1\omega j_n(k_1a)/j_n'(k_1a) \end{aligned} \tag{4}$$

In the above, h is the shell's thickness and a its mean radius; $E_1=Eh/(1-\sigma^2)a^2$ where E is Young's modulus and σ is Poisson's ratio; $\beta^2=h^2/12a^2$ and $\lambda_n=n(n+1)$; ρ_s is the density of the shell's material, and ρ_1 and ρ_2 are the densities of the interior and exterior fluids whose sound velocities are c_1 and c_2; k_1 and k_2 are the acoustic wavenumbers, ω/c_1 and ω/c_2; j_n and $h_n(=j_n+iy_n)$ are spherical Bessel functions; P_n are Legendre polynomials of degree n; the prime on the various quantities denotes differentiation with respect to their arguments. The time-harmonic factor $\exp(-i\omega t)$ is omitted throughout. The geometry of this problem is shown in Figure 1, where in a system of spherical coordinates (R,θ,ϕ) the field quantities of interest are independent of the circumferential angle, ϕ.

The acoustic pressure in the exterior fluid is given by the expression

$$p(r,\theta) = \rho_2 c_2 \omega \sum_{n=0}^{\infty} W_n P_n(\cos\theta) h_n(k_2 R)/h_n'(k_2 a) \qquad (5)$$

and the far-field pressure is obtained from this equation by replacing the spherical Hankel function by its value for a large argument, giving

$$p_f(R,\theta) = \rho_2 c_2 \omega [\exp(ik_2 R)/k_2 R] \times \sum_{n=0}^{\infty} W_n P_n(\cos\theta)\exp[-i(n+1)\pi/2]/h_n'(k_2 a) \qquad (6)$$

3. EXCITATION

In equation (3) the coefficients F_n of the Legendre series expansion of the prescribed radial stress distribution, $F(\theta)$, are defined by the transform pair

$$F(\theta) = \sum_{n=0}^{\infty} F_n P_n(\mu) \qquad (7)$$

$$F_n = (n+1/2)\int_{-1}^{+1} F(\mu) P_n(\mu) d\mu \qquad (8)$$

For the particular case of a radial point force F_0 excitation at $\theta=0$,

$$F(\theta) = F_0 \delta(\theta)/2\pi a^2 \sin\theta$$

$$F_n = (F_0/2\pi a^2)(n+1/2) \qquad (9)$$

Additionally, if the external stress is a constant F_0 over the axisymmetric region defined by $\theta=0$ to α and $\phi=0$ to 2π, then

$$F_n = (F_0/2)[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)] \qquad (10)$$

in which $P_{-1}=P_0$. This is the excitation used in reference [7].

4. EXTERIOR FLUID LOADING

The exact relation between the modal acoustic pressure and the radial displacement at the shell's surface is

$$p_n = \rho c \omega [h_n(ka)/h_n'(ka)] W_n \qquad (11)$$

where ρ and c are the density and sound velocity of the exterior

fluid, and $k=\omega/c$. In the shell's equations (3) and (4) the effect of the surrounding fluid is contained in the coefficient a_{22} of the matrix relation, equation (3),

If $S(n,\omega)$ is defined as the modal receptance of the displacement and pressure at the shell's surface, then its various analytical approximations can be found in Geers & Felippa [6] and Huang & Wang [7] as:

(0) exact

$$S(n,\omega) = \rho c\omega h_n(ka)/h_n'(ka)$$

(1) plane wave approximation, PWA

$$S(n,\omega) = -i\omega\rho c$$

(2) added mass approximation, AMA

$$S(n,\omega) = -\rho a\omega^2/(n+1)$$

(3) doubly asymptotic approximation, DAA1 (12)

$$S(n,\omega) = \rho a\omega^2/[ika-(n+1)]$$

(4) doubly asymptotic approximation, DAA2(M)

$$S(n,\omega) = \rho a\omega^2[ika-(n+1)]/[(n+1)^2-ika(n+1)-k^2a^2]$$

(5) doubly asymptotic approximation, DAA2(C)

$$S(n,\omega) = \rho a\omega^2[ika-n]/[n(n+1)-ika(n+1)-k^2a^2]$$

In Geers & Felippa [6], DAA2(M) is described as a mode derived DAA2, and DAA2(C) is described as a curvature augmented DAA2. Thus, if the terms of equations (12) are used in turn in equations (4), numerical plots of vibration and sound radiation can be used as a test of the range of validity of the various fluid loading approximations of this problem.

5. NUMERICAL RESULTS

(a) General

A Fortran program has been written to calculate radial response and the far-field sound radiation of the point force excited shell. A listing, together with the computer program specifications and test examples, is given in the Appendices. The computational procedure for the far-field sound radiation is straightforward, because the Hankel function in the denominator ensures rapid convergence of the series in equation (6). The series for the displacement in equation (2) converges slowly, so in this case the fluid loading terms for $n \gg ka$ are best computed from their asymptotic form

$$-\rho_2 c_2 \omega k_2 a/(n+1) - \rho_1 c_1 \omega k_1 a/n \qquad (13)$$

with negligible loss of accuracy.

In Figures 2 and 3 which show spectra of the shell's displacement and far-field pressure at $\theta=0$ for the exact and the various exterior fluid approximations, the following constants in SI units were used:

Steel shell: $E=19.5\times10^{10}$ $\sigma=0.29$ $\rho_s=7700$ $a=1.0$ $h=0.01$

Exterior water: $\rho=1000$ $c=1500$ Interior vacuum: $\rho=0$ $c=0$

Shell damping was included in the computations by setting Young's modulus to the complex value $E\equiv E(i-i\eta)$, where the hysteretic loss-factor η was chosen as 0.01. In the plots, the shell's displacement is expressed in dB ref. 1 micrometre, and the pressure is in dB ref. 1 micropascal at 1m. The excitation is a unit force applied at $\theta=0$.

(b) Shell Displacement

The displacement for the EXACT exterior fluid loading is shown in Figure 2A. At 20Hz the anti-resonance is caused by interaction between the n=1 rigid body mode and the forced response of the other modes excited well below their resonant frequencies, which are defined herein as the frequencies at which the modal amplitudes are a maximum. Above 250Hz the shell's resonances dominate in the response, the resonance spacing being a minimum near to 500Hz. Above this frequency the shell becomes increasingly stiff as its mean displacement limits to that of a water loaded infinite plate.

Figure 2B shows the displacement for the PWA fluid loading: this approximation is clearly inadequate at all frequencies. The AMA plot of Figure 2C and the EXACT plot are in good agreement, except for the n=2-4 modes whose AMA amplitudes are too high and too far shifted to higher frequencies. The DAA1 plot of Figure 2D agrees well with the EXACT plot up to 250Hz, when the excessive radiation damping inhibits the formation of resonances. Figures 2E & 2F show that the DAA2(M) and DAA2(C) plots agree with the EXACT plot in all aspects, except that the resonant peaks are underestimated by a maximum of some 4 dB.

(c) Far-field Pressure

The far-field pressure for EXACT exterior fluid loading is shown in Figure 3A. At frequencies up to 200Hz the pressure is close to that obtained from a point force on an infinite plate. Between 200 and 500Hz the spectrum is dominated by a sharp resonant structure. Above 500Hz, the pressure is again close to the pressure of an infinite plate. In this frequency regime the radiation resistance of the modes at resonance is too small for them to contribute to the far-field pressure, which is due to the radiation from modes excited above their resonant frequencies.

Figure 3B shows the sound radiation for the PWA fluid loading: this approximation is poor except at frequencies above 900Hz, where the radiation of the EXACT plot is due to modes radiating with ρc efficiency. The AMA plot of Figure 3C has

much the same shape as the EXACT plot, but the resonant peaks are too high and too far shifted to higher frequencies: also, the spectrum is 3dB too low above 500Hz. Except at very low frequencies the DAA1 plot of Figure 3D is a poor approximation. The levels of the n=3-5 modes of the DAA2(M) plot of Figure 3E are some 10dB too low, but their frequency location is much the same as in the EXACT plot: both low and high frequency regimes provide good agreement with those of the EXACT plot. The DAA2(C) plot of Figure 3F agrees less well with the EXACT plot than does the DAA2(M) plot.

6. CONCLUDING REMARKS

The principal features of the work contained herein are these:

(1) A Fortran computer program which can be used for bench-mark studies of purely numerical formulations of the vibration and sound radiation of submerged elastic structures.

(2) Numerical results of vibration and sound radiation which augment those obtained from other published studies, and reinforce the view that better fluid loading approximations are required in the intermediate frequency regime where resonances occur.

From a practical point of view it may be thought that AMA and DAA2(M) give acceptable results for both vibration and sound radiation, especially if the application is more suited to phenomenological rather than one-to-one modelling, which is usually the case in noise control engineering.

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November 1985

REFERENCES

1. JUNGER M., FEIT D., Sound Structures and Their Interaction, MIT Press 1972.

2. WONG E.H., HAYEK S.I., Vibration and acoustic radiation from point excited spherical shells, The Shock & Vibration Bulletin, No.52 Part 5, May 1982, pages 135-148.

3. JAMES J.H., Intensity vectors of sound scattering by a spherical shell, Admiralty Marine Technology Establishment, Teddington, AMTE(N)TM84080, July 1984.

4. ANSLEY S.J., SKELTON E.A., Sound scattering by a layered sphere, Admiralty Research Establishment, Teddington, AMTE(N)TM85057, May 1985.

5. GEERS T.L., Doubly asymptotic approximations for transient motions of submerged structures, J.Acoust.Soc.Am., 64(5), November 1978, pages 1500-1508.

6. GEERS T.L., FELIPPA C.A., Doubly asymptotic approximations for vibration analysis of submerged structures, J.Acoust. Soc.Am., 73(4), April 1983, pages 1152-1159.

7. HUANG H., WANG Y.F., Asymptotic fluid structure interaction theories for acoustic radiation predictions, J.Acoust.Soc. Am., 77(4), April 1985, pages 1389-1394.

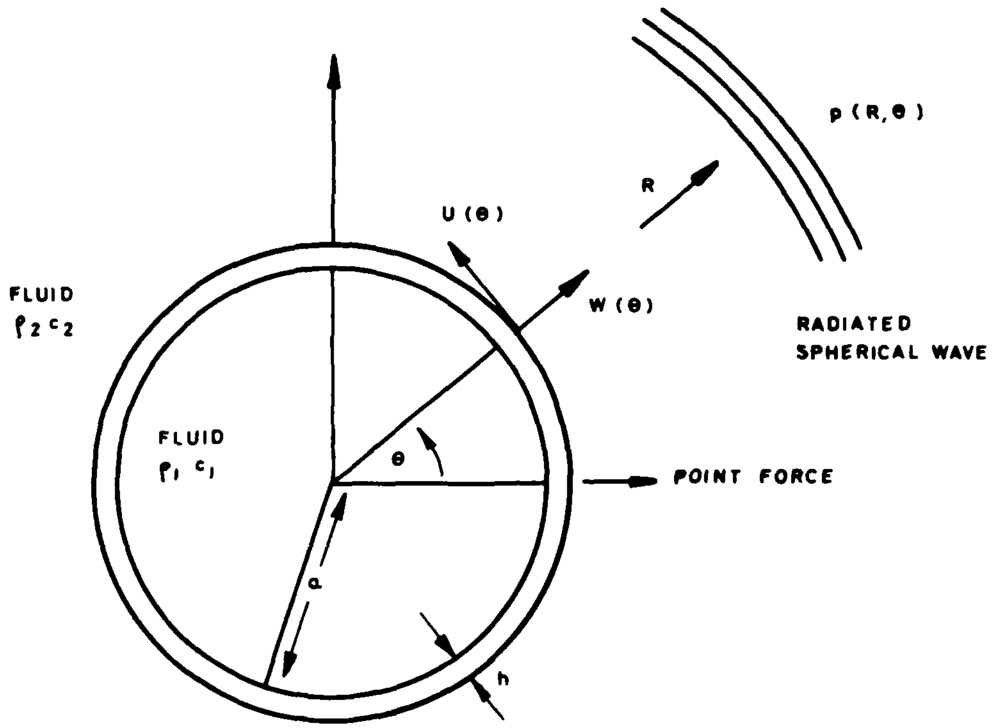

FIG. 1 POINT FORCE EXCITED SPHERICAL SHELL

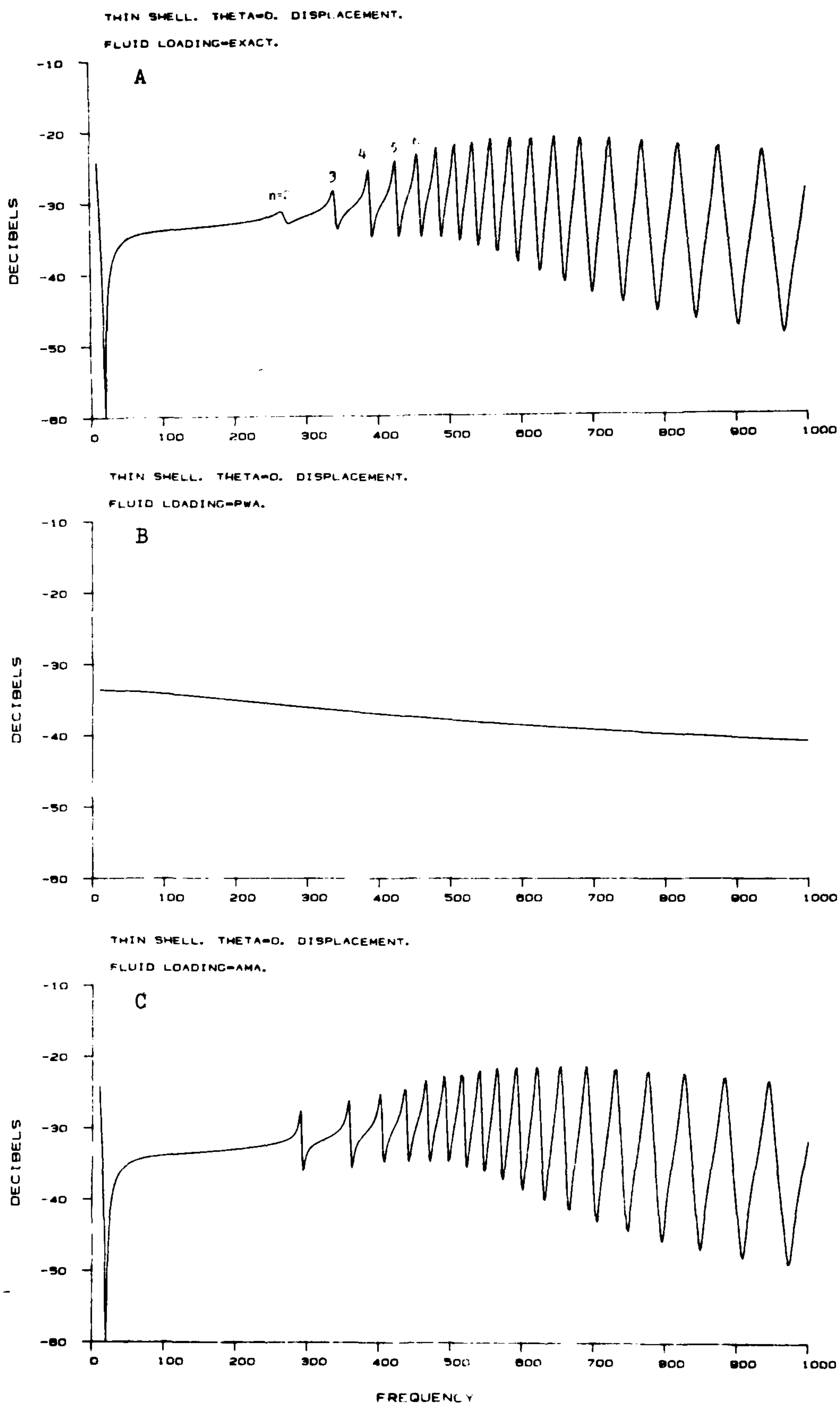

FIG.2 DISPLACEMENT OF SHELL AT θ=0

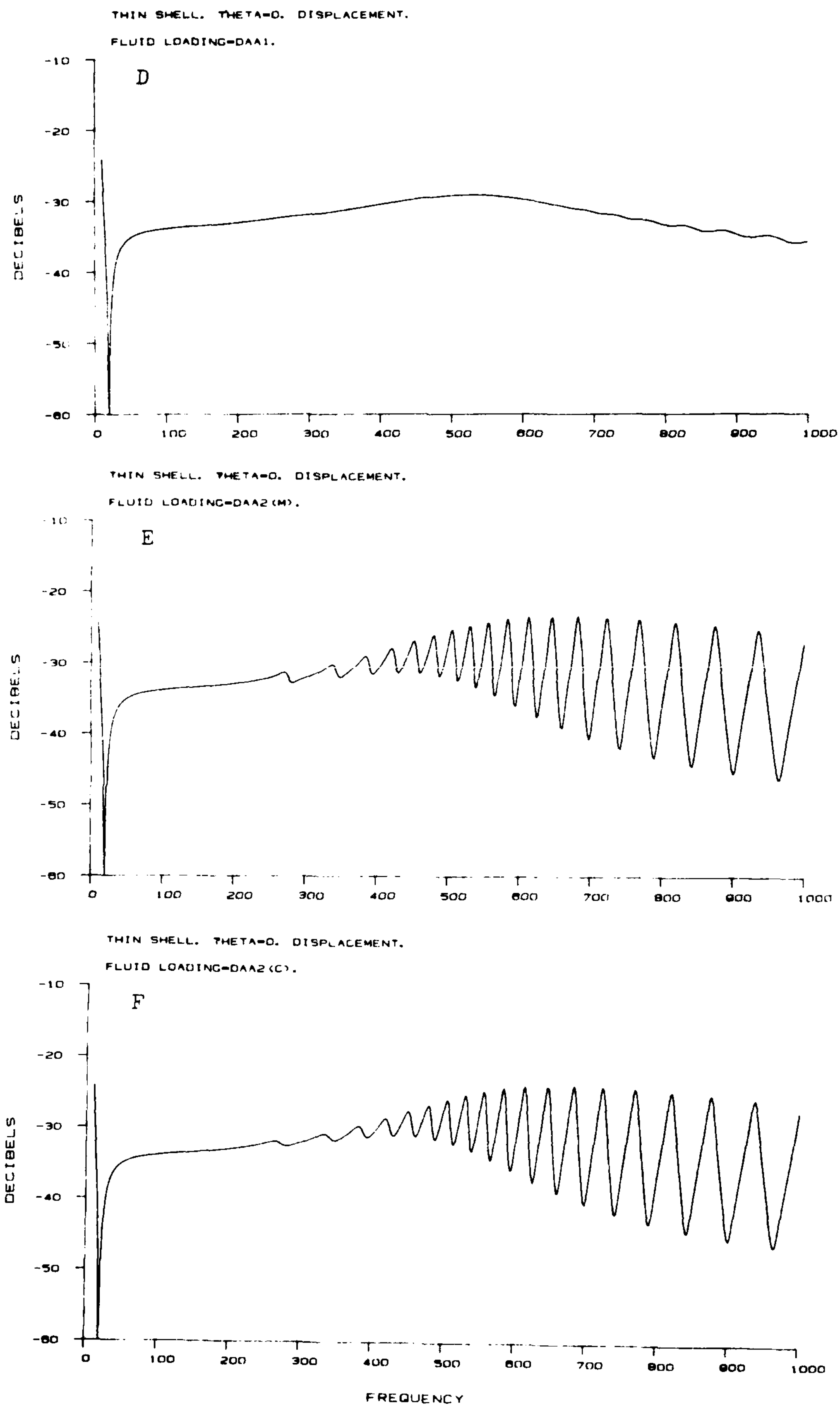

FIG.2 (cont.) DISPLACEMENT OF SHELL AT θ=0

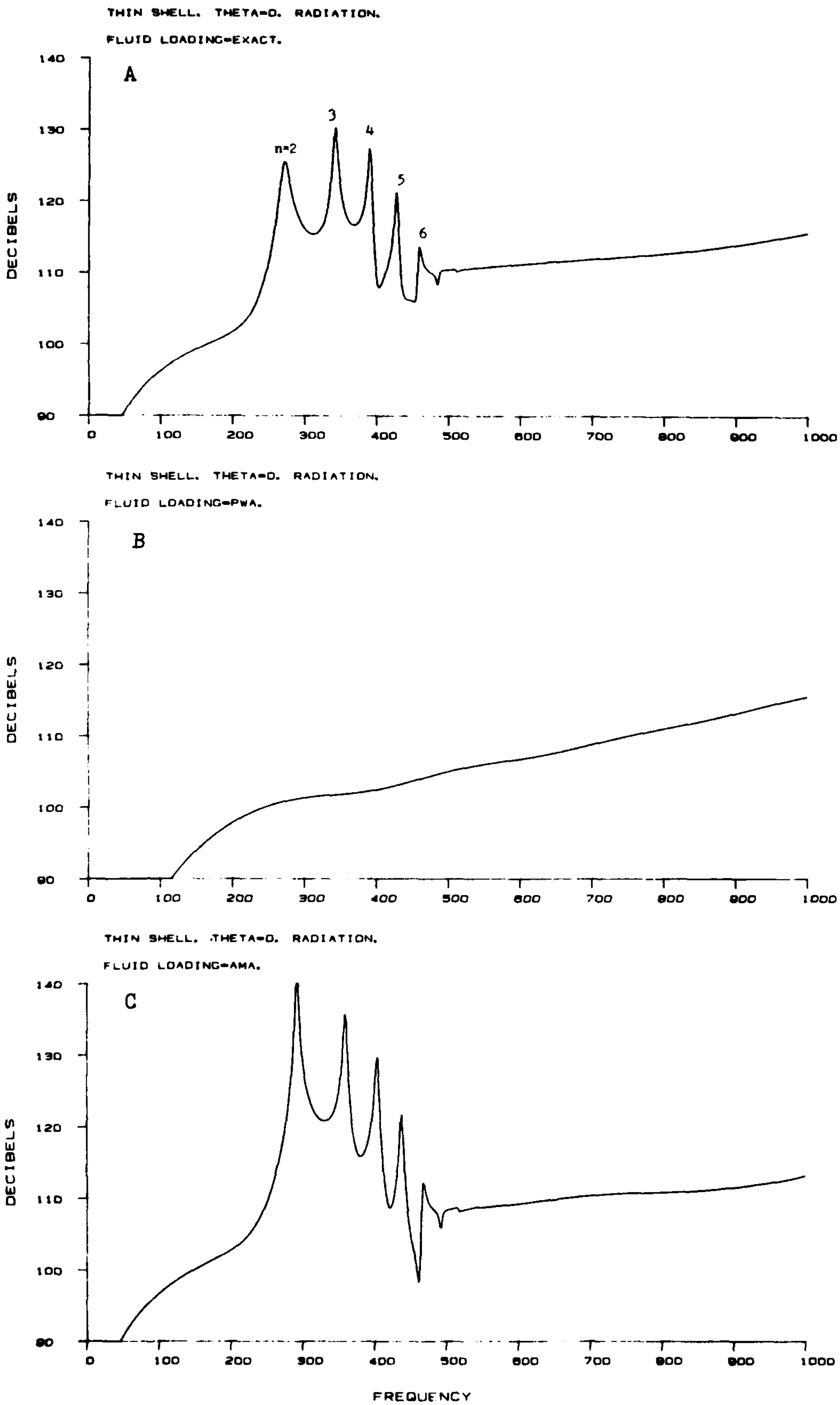

FIG.3 SOUND RADIATION OF SHELL AT θ=0

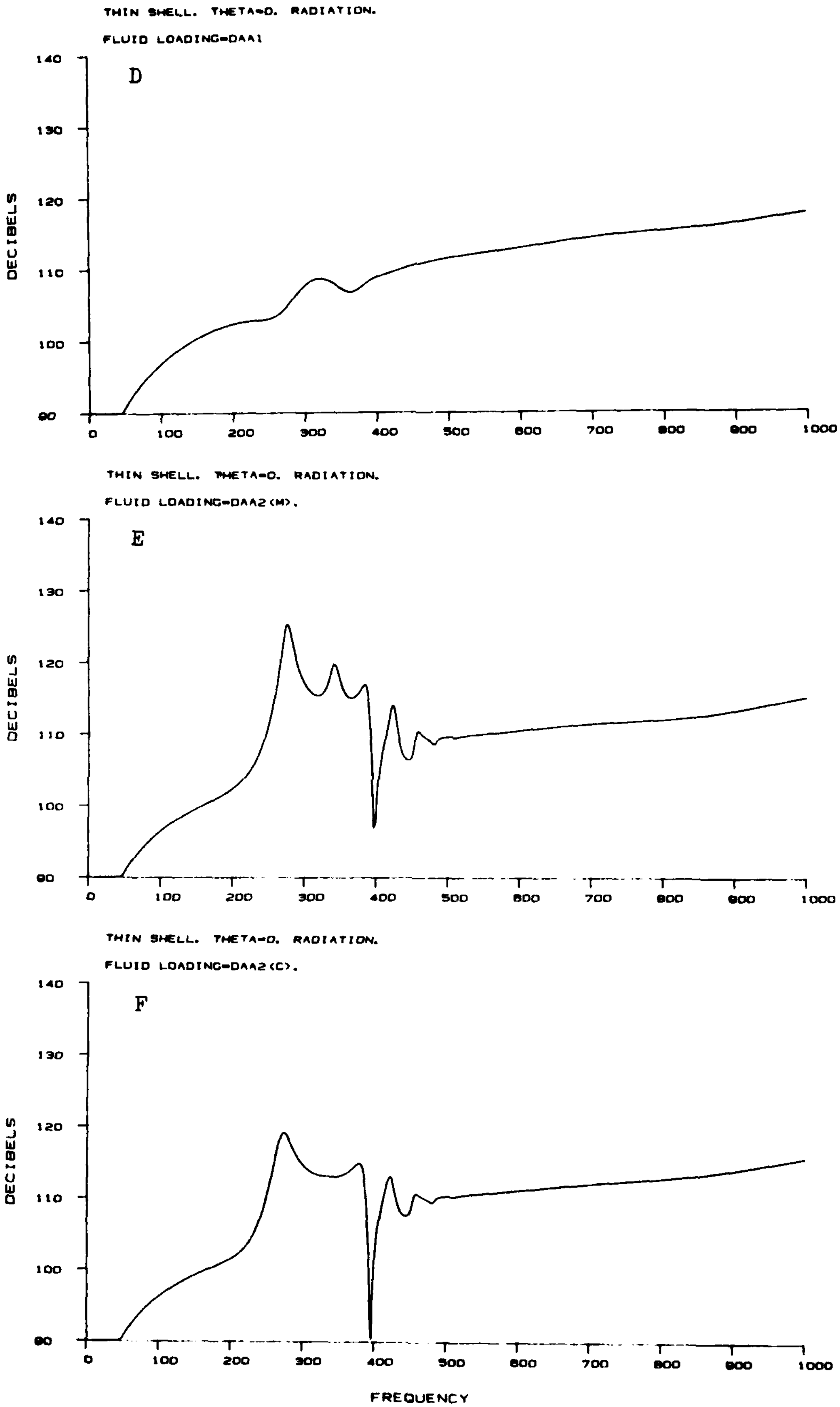

FIG.3 (cont.) SOUND RADIATION OF SHELL AT θ=0

APPENDIX A

PROGRAM SPECIFICATIONS

(a) Data Preparation

Item 01: 6 real numbers, describing the shell.

ES	Young's modulus
VS	Poisson's ratio
RHOS	density
AS	radius
HS	thickness
ETAS	hysteretic loss factor

Item 02: 4 real numbers, describing the fluids.

RHO1	density interior fluid
C1	sound velocity interior fluid
RHO2	density exterior fluid
C2	sound velocity exterior fluid

Item 03: 1 integer, 2 reals and 3 integers.

NF	number of frequencies in linear frequency sweep
FS	start frequency
FM	final frequency
NHS	number of harmonics summed at first frequency
NHM	number of harmonics summed at final frequency
NEX	number of extra harmonics summed for shell response

Item 04: 2 integers, 1 real and 1 integer.

IND1	=0,1,2,3 for computation of far-field pressure, shell displacement, velocity, acceleration
IND2	=0,1,2,3,4,5 for EXACT,PWA,AMA,DAA1,DAA2(M),DAA2(C) computation of exterior fluid loading
THDEG	angle in degrees where response is required, the drive point being at THDEG=0.0
IPLOT	store on disk in block IPLOT. Machine dependent.

(b) Notes on Program

(1) In Item 02, the value of the sound velocity can be set to zero if the density is given as zero.

(2) In Item 03, the start frequency must be greater than zero, a value of about 0.01c/a being suitable. The values of NHS and NHM should be selected by trial and error: values too large will cause overflow. NHM should exceed, by at least 6, say, the number of resonances in the displacement response, and it should also be greater than 5ka. The number of harmonics summed at intermediate frequencies is obtained by linear interpolation.

(3) The integer NEX in Item 03 has no effect if the far-field

pressure is required. It is needed for the shell's response because of the slow convergence of the series. A value such that (NEX+NHM)=90 appears to be suitable. If (NEX+NHM)>98 the array dimensions in SUBROUTINE VIBRAD must be increased. The number of frequencies NF must not exeed 501 unless the array dimensions in the MAIN program are increased.

(4) If IND1=0 in Item 04 the far-field pressure is given in dB ref. 1 micropascal at 1m for unit excitation force, SI units being assumed. If IND1=1,2,3 the shell displacement, velocity or acceleration is given in dB ref. 1 micro-unit for unit excitation force.

(5) The integer IND2 in Item 04 is for the approximation (if any) required for the exterior fluid loading term. IND2=0 gives the exact solution, and IND2=1 to 5 gives the approximations of Section 4. The interior fluid loading is always exact.

(c) Test Example for Sound Radiation

```
19.5E10 0.29 7700.0 1.0 0.01 0.01
0.0 0.0 1000.0 1500.0
3 10.0 1000.0 6 26 0
0 0 0.0 0
```

FREQ	DB	PHASE
10.0	76.7	-88.1
505.0	110.5	-94.1
1000.0	115.6	-88.8

(d) Test Example for Shell Displacement

```
19.5E10 0.29 7700.0 1.0 0.01 0.01
0.0 0.0 1000.0 1500.0
3 10.0 1000.0 6 26 70
1 0 0.0 0
```

FREQ	DB	PHASE
10.0	-24.2	179.8
505.0	-26.2	13.6
1000.0	-28.4	33.8

APPENDIX B

```
C
C.......................................................
C  COPYRIGHT CONTROLLER HER MAJESTY'S STATIONARY OFFICE
C                    LONDON, 1986.
C.......................................................
C
C
      PROGRAM SHRAD2
C
C  JH JAMES. ARE(TEDDINGTON).
C  VIBRATION AND RADIATION OF FLUID FILLED SPHERICAL SHELL.
C
      COMPLEX VALUE
      DIMENSION DB(501),PHASE(501),HZ(501)
      COMMON/SH1/ES,VS,RHOS,AS,HS,ETAS
      COMMON/SH2/RHO1,C1,RHO2,C2,FREQ
      COMMON/SH3/IDV1,IDV2,NHARM,NEX
C
 600  FORMAT(//,1X,'     FREQ','     DB',2X,'  PHASE')
 602  FORMAT(1X)
 604  FORMAT(1X,F9.1,F7.1,2X,F7.1)
C
      IDV1=5
      IDV2=7
      READ(IDV1,*)ES,VS,RHOS,AS,HS,ETAS
      READ(IDV1,*)RHO1,C1,RHO2,C2
      READ(IDV1,*)NF,FS,FM,NHS,NHM,NEX
      READ(IDV1,*)IND1,IND2,THDEG,IPLOT
      WRITE(IDV2,600)
C
C  LOOP FOR EACH FREQUENCY.
      DO 30 I=1,NF
      NFREQ=I
      CON=FLOAT(NFREQ-1)/FLOAT(NF-1)
      FREQ=FS+(FM-FS)*CON
      NHARM=NHS+(NHM-NHS)*CON
      R=1.0
      CALL VIBRAD(R,THDEG,IND1,IND2,VALUE)
      IF(MOD(NFREQ,10).EQ.1)WRITE(IDV2,602)
      XR=REAL(VALUE)
      XI=AIMAG(VALUE)
      DBVAL=10.0*ALOG10(XR**2+XI**2+1.0E-30)+120.0
      PHVAL=ATAN2(XI,XR)*57.296
      WRITE(IDV2,604)FREQ,DBVAL,PHVAL
      HZ(NFREQ)=FREQ
      DB(NFREQ)=DBVAL
      PHASE(NFREQ)=PHVAL
   30 CONTINUE
      IF(IPLOT.LE.0)STOP
C
C  STORE ON DISK FILE FOR PLOTTING. SYSTEM DEPENDENT.
      CALL ASSIGN(1,'DK:FTN1.DAT',0,'OLD')
      DEFINE FILE1(8,4096,U,IVAR)
      WRITE(1'IPLOT)NF,(HZ(I),DB(I),PHASE(I),I=1,NF)
      STOP
      END
C
C
```

```
      SUBROUTINE VIBRAD(R,THDEG,IND1,IND2,VALUE)
C
C  FAR-FIELD PRESSURE, AT DISTANCE R, OR SHELL RESPONSE.
C
      REAL K1,K2
      COMPLEX VALUE,FLUID,WN,CX1,CX2,CX3,CXZERO,CXI
      DIMENSION AJ1(101),AY1(101),AJ1D(101),AY1D(101)
      DIMENSION AJ2(101),AY2(101),AJ2D(101),AY2D(101)
      DIMENSION PN(101),PND(101)
      COMMON/SH1/ES,VS,RHOS,AS,HS,ETAS
      COMMON/SH2/RHO1,C1,RHO2,C2,FREQ
      COMMON/SH3/IDV1,IDV2,NHARM,NEX
C
      NDIM=101
      PI=3.141592654
      CXZERO=(0.0,0.0)
      CXI=(0.0,1.0)
      W=2.0*PI*FREQ
      IF(RHO1.EQ.0.0)GO TO 10
      K1=W/C1
      ARG1=K1*AS
      CALL SPJY(AJ1,AY1,AJ1D,AY1D,NDIM,ARG1)
   10 IF(RHO2.EQ.0.0)GO TO 12
      K2=W/C2
      ARG2=K2*AS
      CALL SPJY(AJ2,AY2,AJ2D,AY2D,NDIM,ARG2)
      ARG3=K2*R
   12 ARG4=COS(THDEG*PI/180.0)
      CALL LEGEND(PN,PND,NDIM,ARG4,NHARM)
C
C  SUM RADIAL SHELL RESPONSE OR FAR FIELD PRESSURE.
      VALUE=CXZERO
      DO 20 NSUM=0,NHARM
      N=NSUM
      FLUID=CXZERO
      IF(RHO2.EQ.0.0)GO TO 22
      CX1=CEXP(CXI*(ARG3-FLOAT(N+1)*PI/2.0))/ARG3
      CX2=CMPLX(AJ2D(N+1),AY2D(N+1))
      CX3=CMPLX(AJ2(N+1),AY2(N+1))
      FLUID=RHO2*C2*W*(CX3/CX2)
      IF(IND2.NE.0)CALL APPROX(FLUID,N,RHO2,C2,W,AS,IND2)
   22 IF(RHO1.NE.0.0)FLUID=FLUID-RHO1*C1*W*(AJ1(N+1)/AJ1D(N+1))
      CALL SHELL(N,W,FLUID,WN)
      WN=WN*FLOAT(2*N+1)/(4.0*PI*AS**2)
      IF(IND1.NE.0)VALUE=VALUE+WN*PN(N+1)
      IF(IND1.EQ.0)VALUE=VALUE+RHO2*C2*W*WN*PN(N+1)*CX1/CX2
   20 CONTINUE
      IF(IND1.EQ.0)RETURN
      IF(NEX.LE.0)GO TO 50
C
C  SUM ADDITIONAL HARMONICS FOR SHELL RESPONSE.
      N1=NHARM+1
      N2=N1+NEX-1
      CALL LEGEND(PN,PND,NDIM,ARG4,N2)
      DO 30 NSUM=N1,N2
      N=NSUM
      FLUID=CXZERO
      IF(RHO2.EQ.0.0)GO TO 32
```

```
      FLUID=RHO2*C2*W*(-ARG2/FLOAT(N+1))
      IF(IND2.NE.0)CALL APPROX(FLUID,N,RHO2,C2,W,AS,IND2)
   32 IF(RHO1.NE.0.0)FLUID=FLUID-RHO1*C1*W*(ARG1/FLOAT(N))
      CALL SHELL(N,W,FLUID,WN)
      WN=WN*FLOAT(2*N+1)/(4.0*PI*AS**2)
      VALUE=VALUE+WN*PN(N+1)
   30 CONTINUE
C
   50 IF(IND1.EQ.2)VALUE=-CXI*W*VALUE
      IF(IND1.EQ.3)VALUE=-W**2*VALUE
      RETURN
      END
C
C
      SUBROUTINE APPROX(FLUID,N,RHO,C,W,A,IND)
C
C  APPROX. FLUID LOADING FOR SPHERICAL SHELL.
C  IND=1/2/3/4/5 FOR PWA/AMA/DAA1/DAA2(M)/DAA2(C)
C
      COMPLEX FLUID,CXI
C
      CXI=(0.0,1.0)
      FN=FLOAT(N)
      FN1=FLOAT(N+1)
      CON=RHO*A*W**2
      AK=W*A/C
      IF(IND.EQ.1)FLUID=-CXI*W*RHO*C
      IF(IND.EQ.2)FLUID=-CON/FN1
      IF(IND.EQ.3)FLUID=CON/(CXI*AK-FN1)
      IF(IND.EQ.4)FLUID=CON*(CXI*AK-FN1)/
     1  (FN1**2-CXI*AK*FN1-AK**2)
      IF(IND.EQ.5)FLUID=CON*(CXI*AK-FN)/
     1  (FN*FN1-CXI*AK*FN1-AK**2)
      RETURN
      END
C
C
      SUBROUTINE SHELL(N,W,FLUID,WN)
C
C  SHELL DISPLACEMENT FOR UNIT RHS.
C
      COMPLEX FLUID,WN,E1,A11,A12,A21,A22,DET
      COMMON/SH1/ES,VS,RHOS,AS,HS,ETAS
C
      W2=W**2
      E1=ES*HS*CMPLX(1.0,-ETAS)/(1.0-VS*VS)
      E1=E1/AS**2
      BETA2=HS**2/(12.0*AS**2)
      DN=FLOAT(N*(N+1))
      X1=VS+DN-1.0
      X2=1.0+VS
      A11=(1.0+BETA2)*X1-W2*RHOS*HS/E1
      A12=BETA2*X1+X2
      A21=DN*A12
      A22=BETA2*DN*X1+2.0*X2-W2*RHOS*HS/E1+FLUID/E1
      DET=A11*A22-A12*A21
      WN=(A11/DET)/E1
      RETURN
```

UNLIMITED

```
      END
C
C
      SUBROUTINE SPJY(AJ,AY,AJD,AYD,NDIM,X)
C
C  EVALUATES THE SPHERICAL BESSEL FUNCTIONS AND DERIVATIVES.
C
      DIMENSION AJ(NDIM),AY(NDIM),AJD(NDIM),AYD(NDIM)
      COMMON/SH3/IDV1,IDV2,NHARM,NEX
C
      N=2*NHARM+1
      IF(N.GE.NDIM)GO TO 70
      AJ(N)=0.0
      AJ(N-1)=1.0E-20
      IL=N-2
      DO 40 I=1,IL
      IP=N-1-I
   40 AJ(IP)=FLOAT(2*IP+1)*AJ(IP+1)/X-AJ(IP+2)
      T=(SIN(X)/X)/AJ(1)
      DO 50 I=1,N
   50 AJ(I)=AJ(I)*T
      AY(1)=-COS(X)/X
      AY(2)=-COS(X)/(X**2)-SIN(X)/X
      N=NHARM
      DO 60 I=1,N
   60 AY(I+2)=FLOAT(2*I+1)*AY(I+1)/X-AY(I)
      AJD(1)=-AJ(2)
      AYD(1)=-AY(2)
      DO 80 I=1,N
      AJD(I+1)=(FLOAT(I)*AJ(I)-FLOAT(I+1)*AJ(I+2))/FLOAT(2*I+1)
   80 AYD(I+1)=(FLOAT(I)*AY(I)-FLOAT(I+1)*AY(I+2))/FLOAT(2*I+1)
      RETURN
   70 WRITE(IDV2,600)
  600 FORMAT(//,1X,'REDUCE FREQUENCY OR INCREASE ARRAY SIZE')
      STOP
      END
C
C
      SUBROUTINE LEGEND(PN,PND,NDIM,X,N)
C
C  LEGENDRE POLYNOMIAL AND DERIVATIVE. MAXIMUM ORDER N.
C
      DIMENSION PN(NDIM),PND(NDIM)
C
      PN(1)=1.0
      PN(2)=X
      NM1=N-1
      DO 10 I=1,NM1
      PN(I+2)=(X*FLOAT(2*I+1)*PN(I+1)-FLOAT(I)*PN(I))/FLOAT(I+1)
   10 CONTINUE
      PND(1)=0.0
      DO 20 I=1,N
   20 PND(I+1)=X*PND(I)+FLOAT(I)*PN(I)
      RETURN
      END
```

END

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